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Steady-state temperature distribution in a doubly connected, orthotropic region with heat generation

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INTRODUCTION

As SHOWN in several recent papers [1–3] the Rayleigh–Schmidt approach is quite convenient when determining approximate analytical solutions to many important solid mechanics problems. The methodology has recently been applied in the case of heat conduction problems [4, 5] and it is extended herewith to the important practical situation of orthotropic regions which appears quite frequently in biomechanics, nuclear engineering, etc.

The title problem is solved by conformally transforming the given region in the z-plane onto an annulus in the ξ-plane. The transformed functional is approximately satisfied using a two-term solution and employing the Rayleigh–Schmidt criterion.

APPROXIMATE SOLUTION

The problem under study is governed by the orthotropic Poisson equation:

$$k_x\frac{\partial^2 T}{\partial x^2}+k_y\frac{\partial^2 T}{\partial y^2}+q = 0 \tag{1a}$$

subjected to the boundary condition

$$T[L_i(x,y) = 0] = 0;\quad i = 1,2 \tag{1b}$$

where $L_i(x, y) = 0$ is the functional relation which defines each boundary of the doubly connected domain.

In order to apply the Rayleigh–Schmidt formulation one expresses (1a) in terms of the equivalent functional

$$J[T] = \iint_D\left[\frac{k_x}{2}\left(\frac{\partial T}{\partial x}\right)^2+\frac{k_y}{2}\left(\frac{\partial T}{\partial y}\right)^2-qT\right]\mathrm{d}x\,\mathrm{d}y \tag{2}$$

subjected to the condition (1b).

Let

$$z = x+yi = f(\xi);\quad \xi = r\,\mathrm{e}^{\theta i} \tag{3}$$

be the mapping function which conformally transforms the given domain onto an annulus in the ξ-plane.

Substituting (3) in (2) results in the transformed functional

$$J[T] = \frac{1}{4}\iint\left\{(k_x-k_y)2Re\left[\left(\frac{\partial T}{\partial \xi}\right)^2\frac{1}{|f'(\xi)^2|}\right]+2(k_x+k_y)\frac{\partial T}{\partial \bar{\xi}}\frac{\partial T}{\partial \xi}\frac{1}{|f'(\xi)^2|}\right\}|f'(\xi)|^2 r\,\mathrm{d}r\,\mathrm{d}\theta -\frac{1}{2}\iint qT|f'(\xi)|^2 r\,\mathrm{d}r\,\mathrm{d}\theta. \tag{4}$$

Taking now a summation of coordinate functions that satisfy the boundary conditions

$$T \cong T_a = \sum_{i=1}^{N} A_i g_i(\xi,\bar{\xi},\gamma) \tag{5}$$

NOMENCLATURE

a_p apothem of a regular polygon
A_1, A_2 arbitrary constant, see equation (8)
J functional
k_x, k_y heat conduction coefficients of the orthotropic medium in the x and y direction, respectively
q heat generation term
r radial variable (transformed plane)
T temperature
z complex variable (real plane).

Greek symbols
α_γ, α_1 coefficients of the coordinate function, see equation (8)
ξ complex variable (transformed plane)
γ optimization parameter
θ angular coordinate (transformed plane).

where in accordance with the Rayleigh–Schmidt technique γ is an optimization exponential parameter, one substitutes (5) in (4) and requires that

$$\frac{\partial J}{\partial A_i} = \frac{\partial J}{\partial \gamma} = 0 \quad (i = 1, 2, \ldots, N). \tag{6}$$

Once the A_is and the γ parameter are obtained one possesses an approximate expression for the temperature distribution.

NUMERICAL RESULTS

In the case of regions of regular polygonal shape with a small concentric hole of radius R_0, expression (3) is given by [6]

$$z = a_\beta A_s \sum a_{1+js}^{1+js} \tag{7}$$

where a: apothem of the polygon, s: degree of polygon, $r_0 = R_0/a_p A_s$ and a_{1+js}: coefficients of the mapping function [6].

For $|\xi| = 1$, equation (7) maps the outer boundary of the doubly connected region and for $|\xi| = r_0$ one obtains the inner, circular boundary (as shown in ref. [6] the approximation obtained is quite good from an engineering viewpoint as long as $R_0/a_p \ll 1$, say $R_0/a_p < 0.40$).

Expression (5) will be taken in the form

$$T_a = (\alpha_\gamma r^\gamma + \alpha_1 r - 1)(A_1 + A_2 \cos 2\theta) \tag{8}$$

where in order to satisfy the transformed boundary conditions $T_a|_{r=1,r_0} = 0$ one must have:

$$\alpha_\gamma = \frac{r_0 - 1}{r_0 - r_0^\gamma}; \quad \alpha_1 = \frac{1 - r_0^\gamma}{r_0 - r_0^\gamma}. \tag{9}$$

Substituting (7) and (8) in equation (4) and requiring that

$$\frac{\partial J}{\partial A_1} = \frac{\partial J}{\partial A_2} = \frac{\partial J}{\partial \gamma} = 0 \tag{10}$$

one obtains an algebraic, nonlinear system of equations in A_1, A_2 and γ. For the present case a simple trial-and-error procedure provided sufficient accuracy for the values of the unknowns.

Numerical results have been obtained for square and hexagonal domains for $R_0/a_p = 0.10$ and 0.20 [the first four terms of equation (7) have been employed in all calculations]. Values of the dimensionless temperature parameter Tk_x/qa_p^2 are depicted in Figs. 1 and 2. The analytical results are in reasonably good agreement with the values obtained by

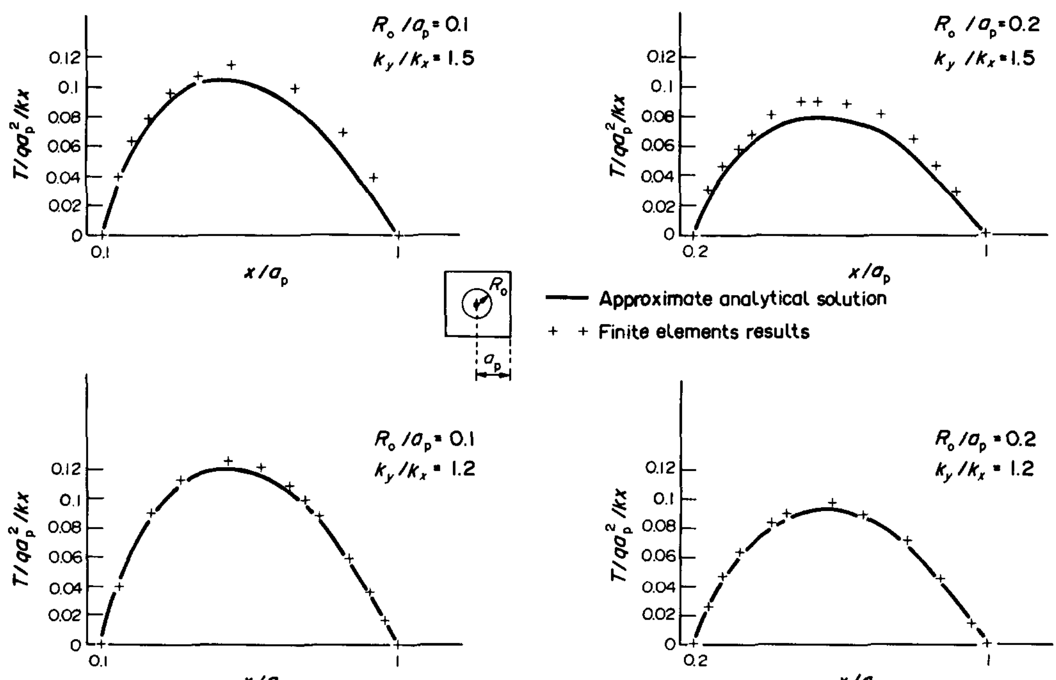

FIG. 1. Dimensionless temperature distribution Tk_x/qa_p^2 ($y = 0$) in the case of a square region with a circular perforation ($R_0/a_p = 0.10, 0.20$ and $k_y/k_x = 1.20, 1.50$).

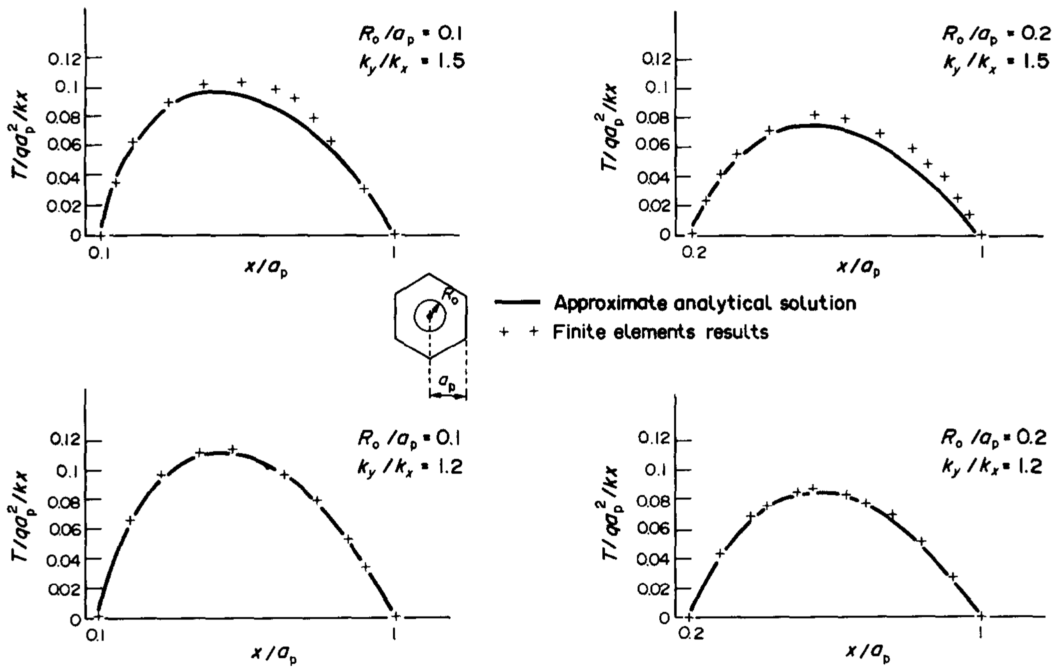

FIG. 2. Dimensionless temperature distribution Tk_x/qa_p^2 ($y = 0$) in the case of a hexagonal region with a circular perforation ($R_0/a_p = 0.10, 0.20$ and $k_y/k_x = 1.20, 1.50$).

means of a finite-element code (this code provides results which differ in less than 1% from exact, analytical solutions).

It is observed that the accuracy of the approximate analytical method improves considerably as the parameter k_y/k_x approaches unity (isotropic case) and also as the order of the polygon increases.

The agreement between the analytical and the numerical results is analogous for other values of the θ coordinate.

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